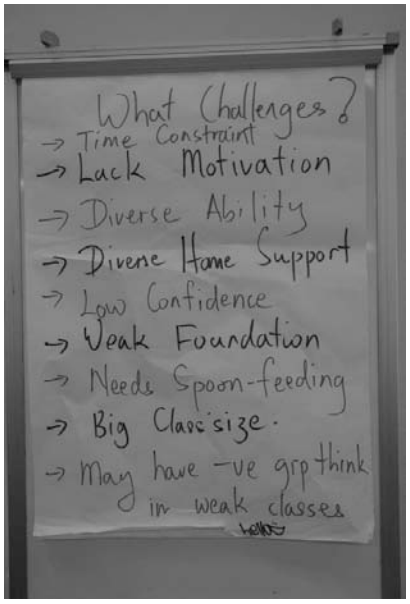
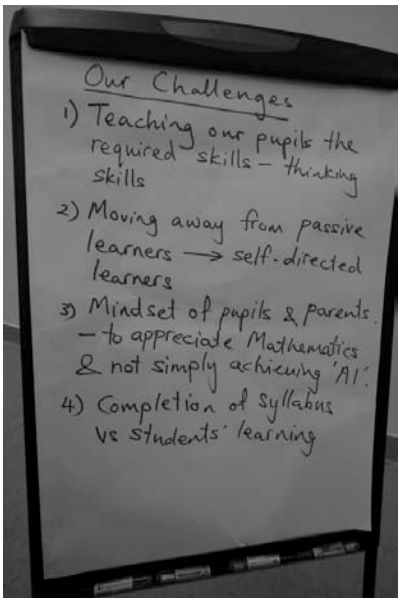
Neriage: An Essential Piece of a Problem-Based Lesson

Teaching through Problem Solving
A Japanese Approach

Akihiko Takahashi, Ph.D.
DePaul University, Chicago IL

Presentation is prepared for the 2008 NCTM Annual Meeting in Salt Lake City, UT on April 11, 2008
by Akihiko Takahashi, DePaul University, Chicago IL

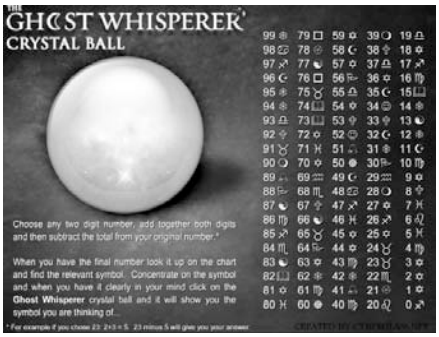




Presentation is prepared for the 2008 NCTM Annual Meeting in Salt Lake City, UT on April 11, 2008
by Akihiko Takahashi, DePaul University, Chicago IL

The Secret of The Crystal Ball

1. Chose any two digit number.
2. Add together both digits.
3. Subtract the total from your original number.
4. When you have the final number look it up on the chart and find the relevant symbol.
5. Concentrate on the symbol and when you have it clearly in your mind.
6. Click on the crystal ball to see the symbol.



http://www.cyberglass.biz/customflash/ghostwhisperer/GWhisperer.swf

NCTM's view of problem solving

1) Problem solving means engaging in a task for which the solution method is not known in advance.
2) Problem solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics program.
3) Choosing worthwhile problems and mathematical tasks
   - There are many, many problems that are interesting and fun but that may not lead to the development of the mathematical ideas that are important for a class at a particular time.

Presentation is prepared for the 2008 NCTM Annual Meeting in Salt Lake City, UT on April 11, 2008
by Akihiko Takahashi, DePaul University, Chicago IL

Problem Solving

Based on Polya's (1945) four phases of problem solving work

A Simplistic Interpretation

Problem Solving as an approach to develop problem-solving skills and strategies.

Problem-solving lessons

For developing problem-solving skills and strategies

often end when each student comes up with a solution to the problem. (show and tell)

Teaching through Problem Solving (PSSM)

Problem solving as a powerful approach for developing mathematical concepts and skills.

Problem-solving lessons

throughout the curriculum in order to develop mathematical concepts, skills, and procedures.

Students' discussion becomes important

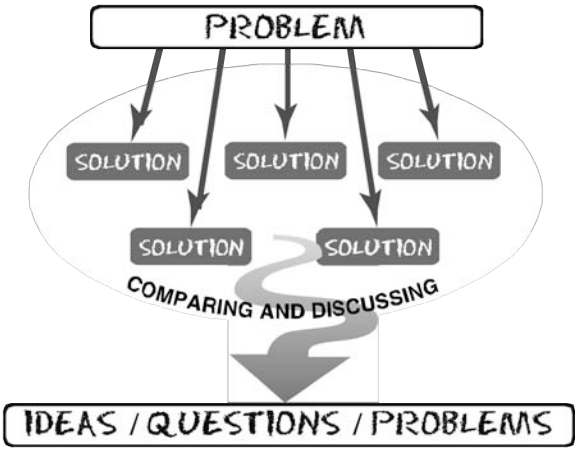
An Example of a worksheet for "Problem Solving"

- **You are selling ice cream from a cart. You sell ice cream bars for $0.75 per bar. Your cost for the ice cream is $0.30 per bar, and your cost for the rental of the cart is $50.**
  a) In a formula, express your total cost *C* as a function of the number of *n of ice cream bars sold.* On graph paper, graph *C* leaving room for negative values on the y-axis.
  b) Express the revenue *R* generated by the sale of ice cream bars as a function of the number *n* sold. Graph on the same graph as in *a*.
  c) Express the profit *P* generated by the sale of ice cream bars as a function for the number *n* sold. Graph *P* on the same graph as in *a* and *b*.
  d) Fine the break even point graphically and algebraically.

Presentation is prepared for the 2008 NCTM Annual Meeting in Salt Lake City, UT on April 11, 2008
by Akihiko Takahashi, DePaul University, Chicago IL

Problem-Based Lesson

- Teaching through Problem Solving -



Presentation is prepared for the 2008 NCTM Annual Meeting in Salt Lake City, UT on April 11, 2008
by Akihiko Takahashi, DePaul University, Chicago IL

Beyond Show and Tell

Neriage

Japanese word for the whole class discussion phase of structured problem solving. It is the core of teaching through problem solving. This happens after students have shared various solution strategies. During this phase, students, carefully guided by the teacher, critically analyze, compare and contrast the shared ideas. They will consider issues like efficiency, generalizability, and similarity to previously learned ideas.

Presentation is prepared for the 2008 NCTM Annual Meeting in Salt Lake City, UT on April 11, 2008
by Akihiko Takahashi, DePaul University, Chicago IL

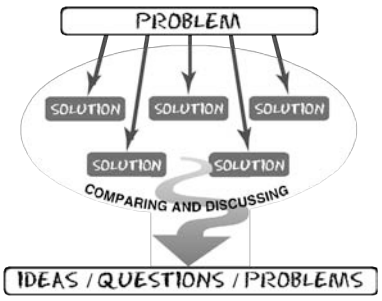
For adequate preparations for leading students to accomplish the goals

1) Anticipating Students' responses

2) Plan for discussion (Neriage) by examining anticipated responses

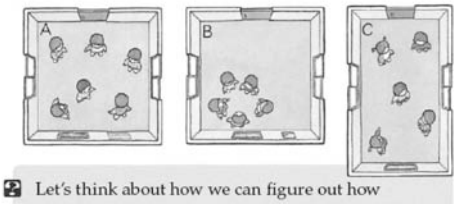
Presentation is prepared for the 2008 NCTM Annual Meeting in Salt Lake City, UT on April 11, 2008 by Akihiko Takahashi, DePaul University, Chicago IL

Japanese Math Textbook Grade 5B p.23-25



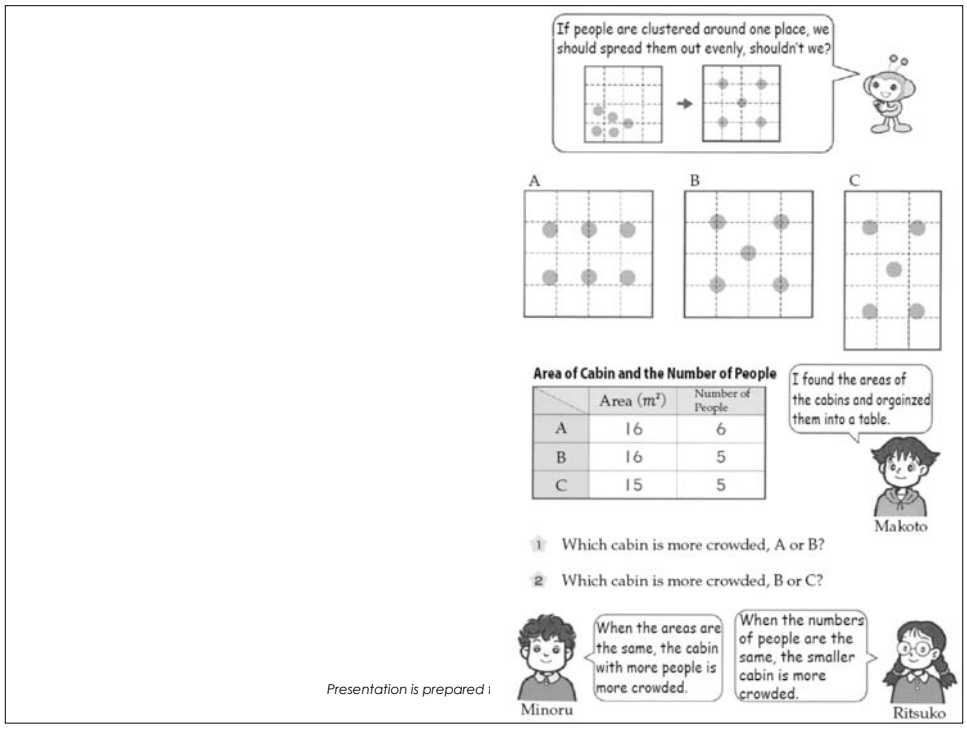
Crowdedness

1 Kiyoshi and his friends will sleep in cabins A, B and C at camp.
Which cabin is the most crowded?



Let's think about how we can figure out how crowded something is!

Presentation is preparec



Presentation is prepared f

From anticipating students' responses

Area of Cabin and the Number of People

	Area (m^2)	Number of People
A	16	6
B	16	5
C	15	5

Cabin A:16÷6=2.666…
Cabin B: 15÷5=3
m^2 / people

Cabin A: 6÷16=0.375
Cabin B: 5÷15=0.333…
people / m^2

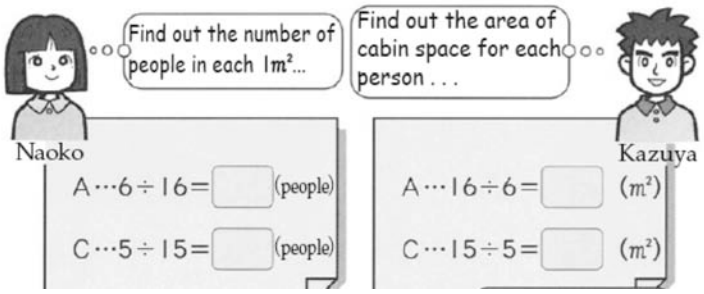
6×5=30
Cabin A:16×5=80
Cabin B: 15×6=90
m^2 / people

16×15=240
Cabin A:6×15=90
Cabin B: 5×16=80
people / m^2

Accuracy, Efficiency, Generalizability

Presentation is prepared for the 2008 NCTM Annual Meeting in Salt Lake City, UT on April 11, 2008 by Akihiko Takahashi, DePaul University, Chicago IL

3 For A and C, the areas as well as the numbers of people are different.
In order to compare the crowdedness of those cabins, what can we do?

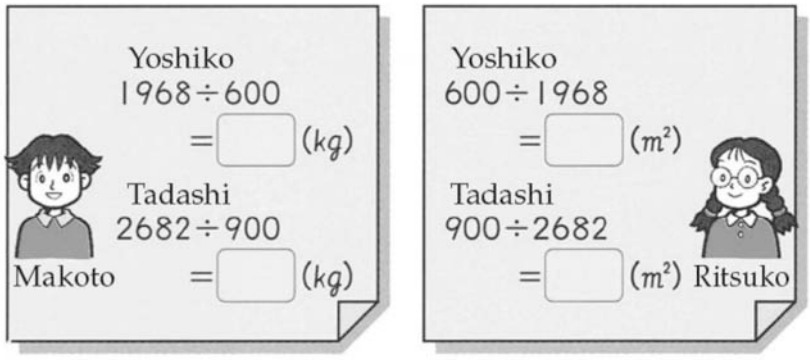


▶ Various per unit quantities

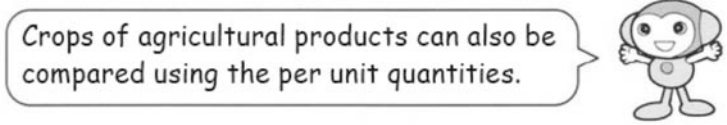
2 At Yoshiko's farm, which is 600m², 1968kg of potatoes were produced.
At Tadashi's farm, which is 900m², 2682kg of potatoes were produced.
Which farm was better at producing potatoes?

Presentation is prepared for the 2008 NCTM Annual Meeting in Salt Lake City, UT on April 11, 2008
by Akihiko Takahashi, DePaul University, Chicago IL

1 Please explain these 2 students' methods.



2 Which farm did better at producing potatoes?



Presentation is prepared for the 2008 NCTM Annual Meeting in Salt Lake City, UT on April 11, 2008
by Akihiko Takahashi, DePaul University, Chicago IL

▶ Population density

3 We researched the areas and populations of Toyama city and Ohita city.
Let's compare the crowdedness of Toyama city and of Ohita city!

Area and Populaion of Toyama city and Ohita city (1995)

	Area (km^2)	Population (people)
Toyama city	209	325303
Ohita city	361	426981

1 How many people are there per $1km^2$ in Toyama city and Ohita city ?
Please round your answers to the second highest place.

Toyama city··· 325303÷209=□ Answer: □ people

Ohita city ··· 426981÷361=□ Answer: □ people

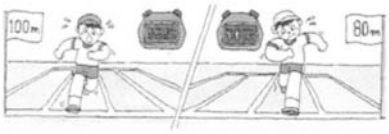
Presentation is prepared for the 2008 NCTM Annual Meeting in Salt Lake City, UT on April 11, 2008
by Akihiko Takahashi, DePaul University, Chicago IL

How to compare speed

1 The table on the right shows the distance and time that Susumu and Kiyoshi ran.
Who ran faster, Susumu or Kiyoshi?

Distance and Time

	Distance (m)	Time (sec.)
Susumu	100	20
Kiyoshi	80	18

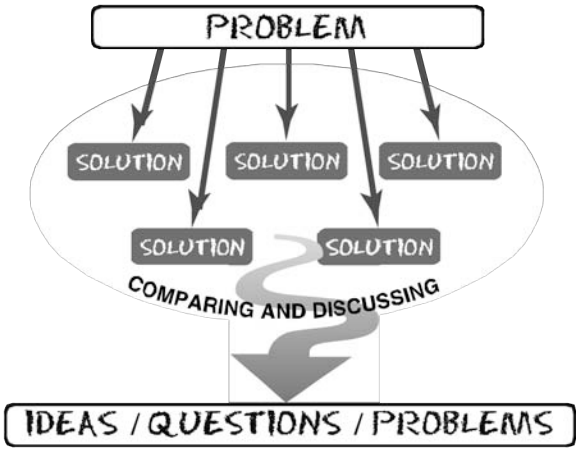


How to express speed

2 The bullet train "Hikari" travels 558km in 3 hours, and "Yamabiko" travels 392km in 2 hours.
Which travels faster?

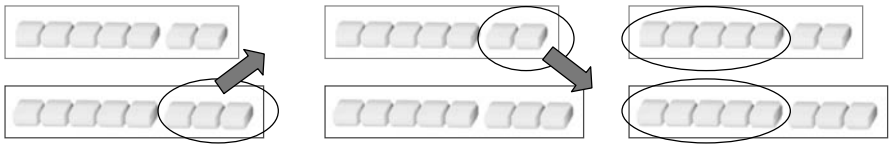
Let's think about how we can express speed!

Instruction as Interaction



Presentation is prepared for the 2008 NCTM Annual Meeting in Salt Lake City, UT on April 11, 2008
by Akihiko Takahashi, DePaul University, Chicago IL

7 + 8



7+8
= 7+(3+5)
= (7+3)+5
=10+5

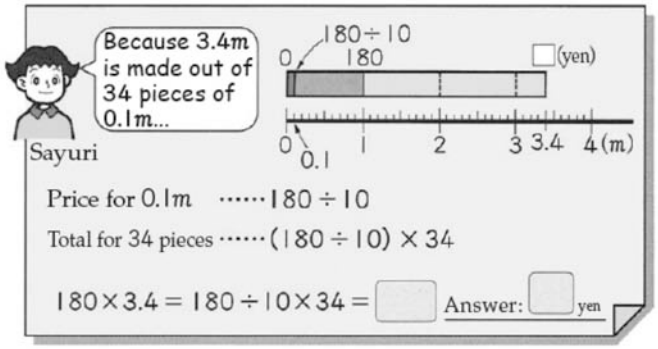
7+8
= (5+2)+8
= 5+(2+8)
=5+10

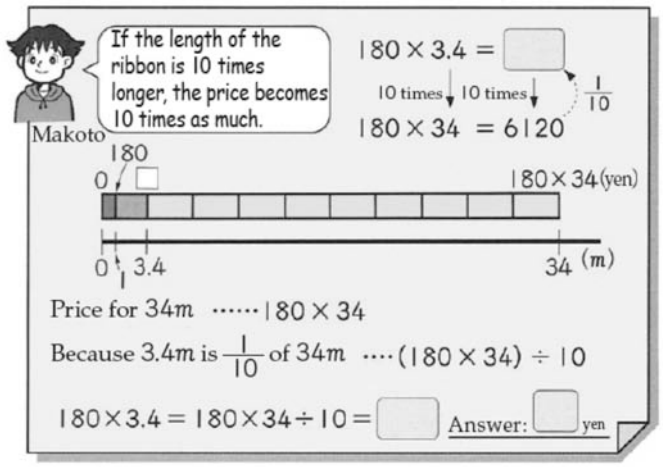
7+8
= (5+2)+(5+3)
= (5+5)+(2+3)
=10+5

4+8 Accuracy, Efficiency, Generalizability

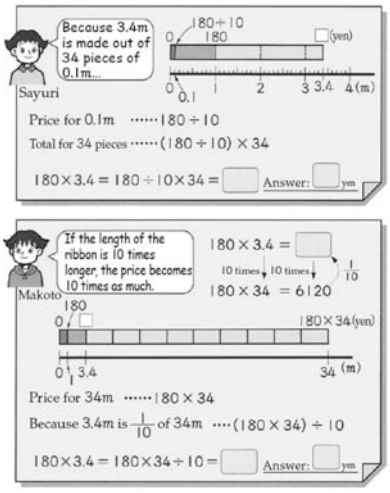
Presentation is prepared for the 2008 NCTM Annual Meeting in Salt Lake City, UT on April 11, 2008
by Akihiko Takahashi, DePaul University, Chicago IL

1 We bought 3.4 meters of ribbon that cost 180 yen per meter. What was the price of the ribbon?





Presentation is prepared for the 2008 NCTM Annual Meeting in Salt Lake City, UT on April 11, 2008
by Akihiko Takahashi, DePaul University, Chicago IL



180×3.4

=(180÷10)×(3.4×10)

=18×34

=612

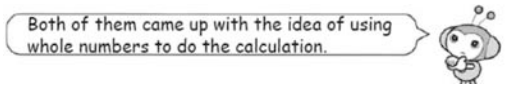
180×3.4

=180×(3.4×10)÷10

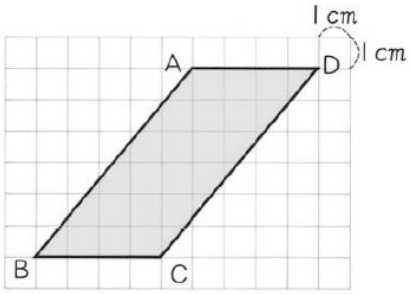
=180×34÷10

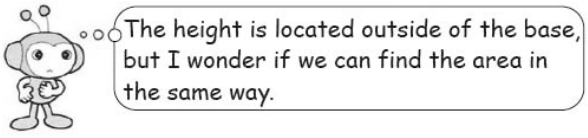
=612

$$\begin{array}{r} 180 \\ \times\ 3.4 \\ \hline 720 \\ 540 \\ \hline 612.0 \end{array} \quad \overset{\times 10}{\Longrightarrow} \quad \begin{array}{r} 180 \\ \times\ 34 \\ \hline 720 \\ 540 \\ \hline 6120 \end{array} \quad \overset{\div 10}{\Longleftarrow}$$



3 Let's think about how to find the area of the parallelogram on the right if we make side BC the base!



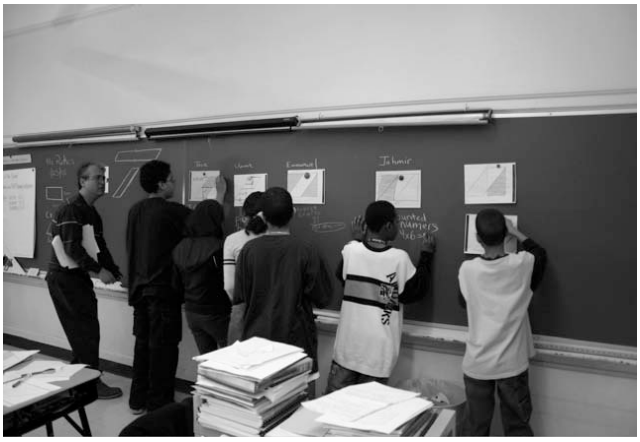


1. Anticipate students solution methods.
2. Derive the formula for finding the area of trapezoid from each anticipated solution.

Students Working on the Problem

Presentation is prepared for the 2008 NCTM Annual Meeting in Salt Lake City, UT on April 11, 2008
by Akihiko Takahashi, DePaul University, Chicago IL

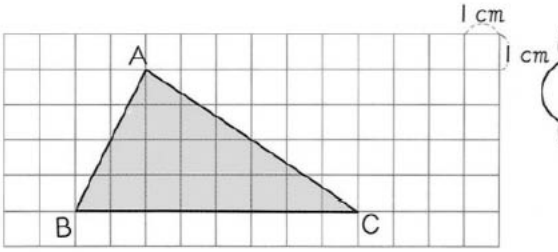
Student Presentation



Presentation is prepared for the 2008 NCTM Annual Meeting in Salt Lake City, UT on April 11, 2008
by Akihiko Takahashi, DePaul University, Chicago IL

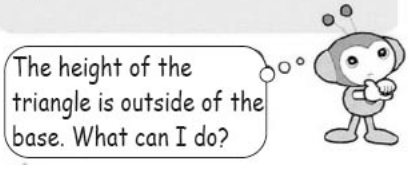
2 Area of Triangles

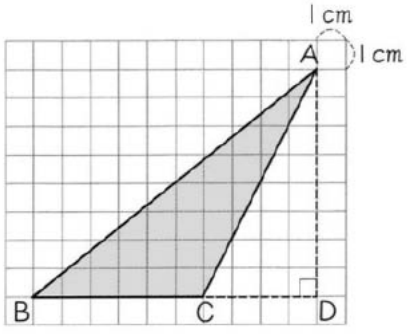
1 Let's think about how to find the area of the triangle below!



Yuji and his friends thought about it in the following ways.

3 Let's think about how to find the area of the triangle on the right when you make side BC the base!

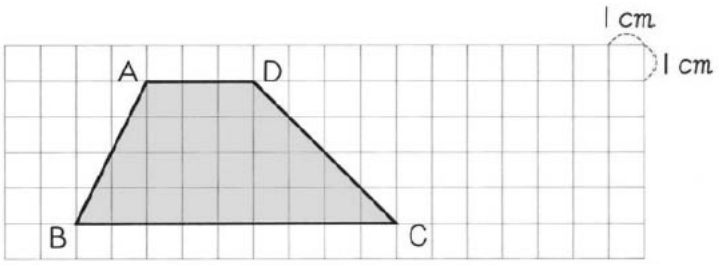




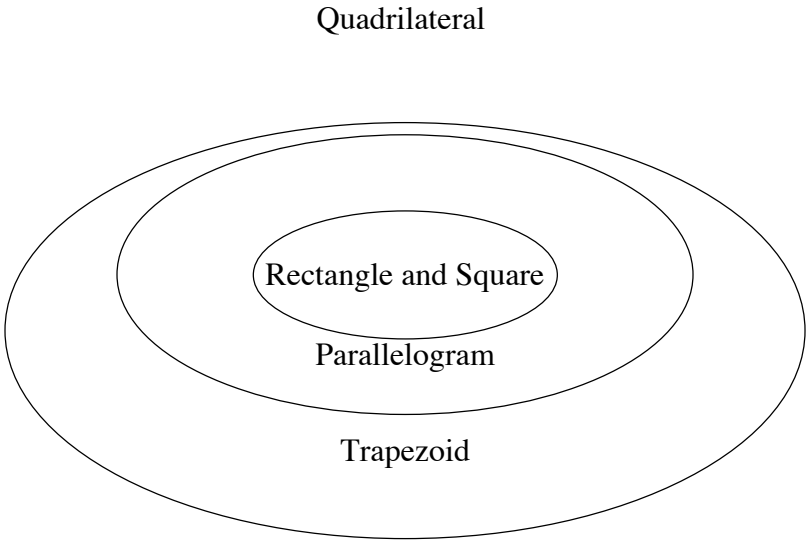
Presentation is prepared for the 2008 NCTM Annual Meeting in Salt Lake City, UT on April 11, 2008
by Akihiko Takahashi, DePaul University, Chicago IL

3 Area of Various Quadrilaterals

1 Let's think about the area of the following trapezoid!



1. Anticipating students' solution methods . Find as many ways as possible.
2. Derive the formula for finding the area of trapezoid from each anticipated solution methods.
3. Discuss how would you like to lead students' discussion in order to help students drive the formula for finding the area of trapezoid.



Presentation is prepared for the 2008 NCTM Annual Meeting in Salt Lake City, UT on April 11, 2008
by Akihiko Takahashi, DePaul University, Chicago IL

http://www.globaledresources.com/

Presentation is prepared for the 2008 NCTM Annual Meeting in Salt Lake City, UT on April 11, 2008
by Akihiko Takahashi, DePaul University, Chicago IL