

Let's think about how to multiply and divide fractions by fractions!

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Takehaya Attached Elementary School,
Tokyo Gakugei University
Grade 6, Class # 1 (21 boys, 20 girls)
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1. Conception of the Activity

(1) Origin of the activity

From 1st grade to 4th grade, students learned the four operations of whole numbers and addition and subtraction of rational numbers. In 5th grade, they learned how to multiply and divide decimals and the meaning and calculation of fraction \times whole number and fraction \div whole number.

After the students learned about multiplication and division of a decimal by a decimal, I asked them, "What do you think that we should study next?" the students replied, "We want to do multiplication of fractions when the multiplier is a fraction and division of fractions when the divisor is a fraction because we haven't learned them yet." So, based on the students' recommendations, we decided that they will study fraction \times fraction and fraction \div fraction next.

The students are highly motivated to learn how to calculate all types of numbers (whole numbers, decimal numbers, and fractions) using the four operations.

(2) Wishes of the teacher

I would like to support my students' desire to want to be able to calculate all types of numbers using the four operations, and I truly hope that they master these skills. Furthermore, the students are beginning to become more aware of how their own ideas are received by their peers and whether or not their ideas are better than other ideas. This awareness will become even more evident when they begin discussing and trying to determine things such as "which solution is easier to understand," or whether or not "the ideas always can be applied to other numbers" by looking at the many different ideas and solution methods that are presented. In addition, the students are trying to relate their own thinking and ideas to their peers' thinking and ideas. I hope to develop this type of thinking in my students.

In this activity, I would like to bring out the desire in my students to calculate all types of numbers using the four operations as well as develop their desire to relate to their peers' thinking and ideas. In order to do this, I would like to prepare a situation where the students can feel a sense of accomplishment when they complete the calculation using the four operations and learn from their peers' thinking and ideas while still valuing their own ideas.

I would like my students to understand that calculation methods are supported by the characteristics of numbers and the properties of operations. I would like to provide this kind of activity to my students because this is a long-held belief of mine.

Also, at the end of this activity, I would like to summarize the number concepts (positive rational numbers and 0) that students have studied thus far. By asking the question “What are we able to do by using numbers?” the students will go back and read their elementary mathematics textbooks. Then, I would like to help them to once again recognize and understand that we can use numbers to express quantities and order of objects, measure quantities, and calculate.

2. Plan of the Unit

Let’s think about how to calculate fraction x fraction and fraction ÷ fraction! (6 lessons)

Goals

- Think about how to calculate fraction x fraction and fraction ÷ fraction by using previously learned calculation methods, properties of fractions, and properties of multiplication and division.
- Understand the method of calculation for fraction x fraction and fraction ÷ fraction
- Understand that when we use numbers, we can express quantities and order of objects, measure quantities, and calculate.

Activity Plan

Phase 1: “Let’s think about how to multiply a fraction by a fraction!”

1st and 2nd lesson: “Let’s think about how to calculate $\frac{2}{5} \times \frac{3}{4} = \frac{3}{10}$!”

(A) $0.4 \times 0.75 = 0.3$ can be expressed with fractions as $\frac{2}{5} \times \frac{3}{4} = \frac{3}{10}$. So by thinking about the

product of the fraction calculation, $\frac{3}{10}$, students can think about how to calculate fraction x fraction.

- (B) Students will notice that product of two fractions can be found by multiplying both numerators and both denominators by looking at several calculation methods. In addition, they will learn the method of calculation.
- (C) Students can think about how to calculate by using the calculation methods, properties of fractions, and properties of multiplication and division that they learned previously.
- (D) Students will confirm whether this method can work for calculations involving fractions that cannot be expressed as decimals, as well as understand that it can be done.
- “I wonder if we can change the calculation to fractions x whole number.”
 - “There are several ways to calculate but all of them can be transformed to a math sentence that multiplies the both numerators and both denominators.”
 - “I wonder if we can use the same method for fractions that cannot be expressed as decimals.”

Phase 2: “Let’s think about how to divide a fraction by a fraction!”

1st and 2nd lesson: “Let’s think about how to calculate $\frac{9}{20} \div \frac{3}{5} = \frac{3}{4}$!” (This lesson)

◇ Since $0.45 \div 0.6 = 0.75$ can be expressed with fractions as $\frac{9}{20} \div \frac{3}{5} = \frac{3}{4}$, students can think about how to divide fractions by thinking about the quotient, $\frac{3}{4}$.

◇ Students can think about how to calculate by using the calculation methods, properties of

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- fractions, and properties of multiplication and division that they learned previously.
- ◇ By examining several calculation methods that are presented, students can think about a method that can be applied any time.
 - ◇ Students will confirm whether this method can work for calculations involving fractions that cannot be expressed as decimals and understand that it can be done.
 - “I wonder if we can change the calculation to fraction \div whole number?”
 - “There are several ways to calculate but all of them can be transformed to a math sentence that multiplies by the inverse fraction of the divisor.”
 - “I wonder if we can use the same method for fractions that cannot be expressed as decimals.”

Phase 3: “Let’s summarize what we can do with numbers!”

1st lesson: “Let’s think about how to multiply a whole number by a fraction and divide a whole number by a fraction!”

- (E) Students will think about how to calculate whole number \times fraction and whole number \div fraction by going over what we know about multiplication and division calculations.
- (F) Students will feel a sense of accomplishment for being able to calculate all of the different kinds of numbers they learned with the four operations.
 - “We have not learned how to multiply a whole number by a fraction and divide a whole number by a fraction, so let’s think about those calculations!”
 - “If we change the whole numbers into fractions we can do the calculation just like we did with fraction \times fraction and fraction \div fraction.”
 - “Now we can calculate with the four operations with whole numbers, decimals, and fractions.”

2nd lesson: “By reviewing our math textbooks from 1st to 6th grades, let’s summarize what we can do with numbers!”

- (G) Students will understand that when we use numbers, we can express quantities of objects, express order of objects, measure quantities, and calculate.
 - “We can count the number of objects.”
 - “We can measure length, weight, etc.”
 - “We can express ratio.”
 - “We can calculate”

3. About this lesson

(1) Goal of the lesson

- Students will think about how to calculate by using the calculation methods, properties of fractions, and the property of division (i.e., when you multiply or divide both the dividend and the divisor by the same number, the quotients will remain the same) that they learned previously.
- To understand the ideas and solution methods presented by other students and think about which calculation method can be used in every case.

(2) Lesson Development

Students' main activities and anticipated reactions	◇ Instructional support ◆ Observation points
<p>○ Calculate $0.45 \div 0.6$.</p> <ul style="list-style-type: none"> • Calculate by using an algorithm • By using a property of division (when you multiply both the dividend and the divisor by the same number, the quotients will remain the same), change it to $45 \div 60$, then calculate. • Understand the result of the calculation: $0.45 \div 0.6 = 0.75$ <p>○ Think about why the quotient of $\frac{9}{20} \div \frac{3}{5}$ becomes $\frac{3}{4}$ by expressing $0.45 \div 0.6 = 0.75$ with fractions.</p>	<p>◇ Because we can express a number using both decimals and fractions, and students have learned decimal \div decimal calculation, the teacher will suggest to the students: "We can express fraction \div fraction using decimal \div decimal, so let's calculate by using decimal \div decimal since we already know how to do that!"</p> <p>◇ If the students do not come up with a calculation idea using the property of division (when you multiply or divide both the dividend and the divisor by the same number, the quotient will remain the same), the teacher will ask the students: "Is there any way we can change the calculation to whole number \div whole number?" Write down the property on the blackboard so students can use it to think about solving the problem on their own.</p> <p>◇ Tell students: "Let's express it using fractions!" Ask the students to simplify the fraction.</p>
<p>Let's think about how to calculate $\frac{9}{20} \div \frac{3}{5}$!</p>	<p>◆ Can they think about how to do the calculation by utilizing the properties of fractions and the property of division?</p>
<p>○ Think about how to calculate on their own:</p> <p>(A) When we calculate fraction \times fraction, we multiply both numerators and both denominators. Since this is a division problem, we can divide both numerators and both denominators. $\frac{9 \div 3}{20 \div 5} = \frac{3}{4}$</p> <p>(B) Using a property of division (when you multiply both the dividend and divisor by the same number, the quotients will remain the same) change the division into a fraction \times whole number to pursue the calculation.</p>	<p>• If the teacher finds the students doing the calculation $\frac{9}{20} \div \frac{3}{5} = \frac{9}{20} \div 3 \div 5$ in order to change the calculation to a whole number calculation, and ending up with $\frac{9}{20} \div 3 \div 5 = \frac{3}{100}$, ask them to think about a different method by saying "It looks like the quotient is different from the quotient we got from the decimal calculation."</p> <p>• If the teacher finds students who are having a hard time coming up with an idea,</p>

$$\frac{9}{20} \div \frac{3}{5} = \frac{9}{4} \div 3 = \frac{3}{4}$$

(C) Using a property of division (when you multiply both the dividend and the divisor by the same number, the quotients will remain the same) change the division into whole number \times whole number to pursue the calculation.

$$\frac{4}{20} \div \frac{3}{5} = 9 \div 12 = \frac{9}{12} = \frac{3}{4}$$

○ Present ideas about how to calculate and understand each others' ideas.

- (B) and (C) above use the same property of division
- (A) might not work with different fractions.

For example: $\frac{3}{4} \div \frac{2}{5}$

○ Check to see if $\frac{3}{4} \div \frac{2}{5}$ can be solved using the ideas from (A) to (C).

- Method (B) works well.

$$\frac{3}{4} \div \frac{2}{5} = \frac{15}{4 \div 2} = \frac{15}{8}$$

- Method (B) works well. If I write the calculation process clearly:

$$\frac{3}{4} \div \frac{2}{5} = \frac{(3 \times 5)}{4} \div 2 = \frac{(3 \times 5)}{(4 \times 2)}$$

- Method (C) works well.

$$\frac{3}{4} \div \frac{2}{5} = 15 \div 8 = \frac{15}{8}$$

- Method (C) works well if I write the calculation process clearly:

$$\frac{3}{4} \div \frac{2}{5} = (3 \times 5) \div (2 \times 4) = \frac{(3 \times 5)}{(2 \times 4)}$$

- Method (A) works well, too. If we can find a multiple of the numerator and the

denominator $\frac{3}{4}$ and $\frac{2}{5}$ it works. First we

can do: $\frac{3}{4} = \frac{6}{8} = \frac{30}{40}$ Then, do the

$$\frac{30}{40} \div \frac{2}{5} = \frac{(30 \div 2)}{(40 \div 5)}$$

- We can write the calculation process of (A) more clearly:

$$\frac{(3 \times 2 \times 5)}{(4 \times 2 \times 5)} \div \frac{2}{5} = \frac{(3 \times 5)}{(4 \times 2)}$$

- When we write the calculation process of (B), (C), and (A) clearly, all of the math

encourage them by saying "What did we do when we did the calculation of fraction \times fraction?" By helping students recall this, you are helping them to think and come up with at least one idea that uses the property of division.

- ◆ Can students understand the solution methods presented by their peers? Can they see the similarities and differences in the ideas by comparing them to their own ideas?

◇ If the students never come up with an idea for exploring other fractions besides $\frac{9}{20} \div \frac{3}{5}$, ask

them "Do you think the methods we looked at today always work for any fraction \div fraction problem?"

- ◆ Are they actively investigating whether the methods they find during this lesson can be used for other fraction \div fraction calculations?

◇ When a student presents method (B), if the student did not clearly explain the process of calculation by using a non-described number such as 15 of $\frac{15}{8}$, the teacher should ask a question such as

"Where does this 15 come from?" and clarify that the 15 is 3×5 . Also, make sure to record it on the blackboard.

- ◆ Are students actively looking for a common calculation procedure by looking at mathematical expressions (A) to (C)?

◇ If students are having difficulty finding the common calculation procedure, the teacher will ask "Are there similarities in the calculation procedures when we rewrote them more clearly?"

◇ If students can't see the part about inverting the denominator and the numerator of the divisor and then multiplying the dividend, ask "Where can you find the 3 in the first math expression?" to help them notice the idea.

◇ If the fractions involved in the second problem,

such as $\frac{3}{4} \div \frac{2}{5}$, can be expressed as decimals

ask the students, "Can we divide fractions without changing them into decimals?"

sentences involve multiplication by the
inverse of $\frac{2}{5}$.

○ Journals writing

- I understand that if we can change the problem into a calculation that we learned before, we can solve it.
- I think that the properties of division are very important.
- I was surprised that we can do the problem using method (A).
- I felt happy that we found the calculation method for fraction \div fraction