

Grade 5 Mathematics Lesson Plan

July 4 (Wed.), 2007, Period 5 Setagaya Kyuden Elementary School

Grade 5 Class 1 (40 students)

Teacher: Takehito Tsuruoka

1. Title of the unit: Secrets of numbers in a table

2. Goals of the unit

Students will discover intriguing patterns in numbers represented in a chart by decomposing and re-arranging the numbers as they determine the total of the chart in creative ways.

3. Evaluation standards

Interest, desire, and attitude	<ul style="list-style-type: none">• Students will try to find simpler ways to calculate the total in a number chart
Mathematical thinking	<ul style="list-style-type: none">• Students will be able to explain the reasoning behind mathematical expressions used to find the total in number chart.• Students will be able to use multiplication by multiplying the numbers arranged in patterns and making them the same numbers.
Representations and procedures	<ul style="list-style-type: none">• Students will be able to write math sentences to find the sum of the numbers arranged in a pattern, and they will be able to determine the total accurately.
Knowledge and understanding	<ul style="list-style-type: none">• Students will understand how to find the total of the numbers in a number chart.

4. Main points of the lesson

The theme of the Mathematics Division of the Setagaya District Elementary Lesson Study Group is “savoring the joy of thinking.” We analyzed children’s learning experiences that may help them move closer to this goal. We believe that the following 4 types summarize such learning experiences.

- Engaging in learning tasks with their own initiative.
- Pursuing better approaches.
- Comparing and relating various ideas and reasoning processes.
- Exploring possible extensions.

What is common in these types of learning experiences is that they are all motivated by students’ own desire to understand something better. Students will work with a simple number chart in today’s lesson. We would like students to develop a desire to

figure out a better way to calculate the total and an attitude to explore how they can use the ideas from today's lesson in other situations.

(1) Engaging in learning tasks on their own initiative

In the introduction of today's lesson, students will use calculators to find the total of 9 numbers taken from a number chart. The teacher will mentally calculate the sum, but he will be able to obtain the result much more quickly than the students with calculators. Students who simply calculated the total without thinking too much will naturally ask, "How did you do that?" Their own question will then become the motivating force for students to think further. There is much wonder and beauty in numbers based on their patterns. By experiencing one such instance themselves, students will want to pursue the joy of mathematics and thinking.

(2) Pursuing better approaches

The main task during the individual problem solving time is for students to devise a way to find the total. We believe that students will try to devise a way without being told by the teacher to do so. This problem involves a "secret" that is often used in elementary school mathematics lessons. We believe it is important for students to attempt to figure out that secret by trying out different ideas.

The problem in today's lesson involves the idea of average (arithmetic mean). However, 5th grade students have not studied the idea of average yet. Therefore, it will be difficult for them to use the idea of "evening out" with anticipation and understanding to solve this problem.

Therefore, we will provide the following 9 numbers:

11	12	13
21	22	23
31	32	33

With these numbers, the difference between the adjacent numbers in each row is always 1, and in each column, the difference is always 10. Because students can easily notice these patterns, we believe they can think about the idea of modifying the numbers to make the same number.

These numbers will make it easier for some students to think about separating the ones and the tens digits. From that point, we hope to connect the idea, "when there are several groups of the same amount, we can use multiplication," to the goal of this lesson.

(3) Comparing and relating various ideas and reasoning processes

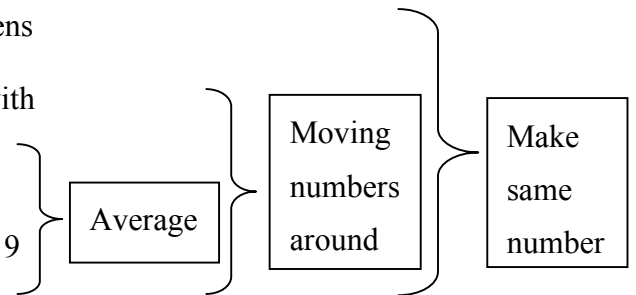
We will begin the sharing and discussion time by first using mathematical expressions to explain their thinking processes. At that point, it is important for students to figure

out how the person thought about the problem by looking at their mathematical expressions.

That is when students employ various thought processes. Where is 21? Why $\times 3$? Etc. The role of the teacher during the sharing period is to help students make connections between mathematical expressions and ways of thinking about the problem. By listening carefully to a child's comments and explanation, the teacher can record important ideas on the blackboard. By observing different shared ideas, students will develop their capacity to compare various ideas.

5 anticipated responses for today's problem

- Calculate by separating the ones and the tens digits.
- Make 44 by pairing one "large" number with one "small" number in the chart.
- Averaging each row.
- Averaging each column.
- Based on (b) ~ (d), find the average of the 9 numbers.



Students will be asked to think about what is common in these strategies and how they are different from each other. They will also examine whether any approach is better than the others. Students can develop anticipation and foresight only after they are able to organize the connections among different strategies. The beginning of savoring the "joy of thinking" is to have anticipation and foresight. Therefore, the teacher will make sure that the sharing and discussion period will not end up as just simply sharing various solutions. Rather, the teacher will provide whatever support necessary to orchestrate the discussion so that students can experience the merits of each approach and recognize the connections among various approaches.

(4) Exploring possible extensions

A rather simple number chart will be used in today's lesson, and at the end of the lesson, it is hoped that students will realize that "if we even out the numbers, then we can use multiplication to find the total very easily." If we stop at this point, students' thinking also stops here. However, it is important for students to further broaden and deepen their thinking by starting with the question, "Why can we do this?" and continuously asking, "Can we always do this?", "Can we use different numbers", "Can we use a different number chart?" etc.. It is our goal to nurture our children so that when we ask at the end of the lesson, "Can we...?" they will say, "I want to try!"

5. Flow of the unit

Lesson	Goals	Instructional Activity	Students' questions
1 (today)	<ul style="list-style-type: none"> Students will be able to determine the sum of the numbers selected from a number chart by identifying patterns in the number chart and moving numbers around as appropriate. Students will find the number charts intriguing and fun as they calculate the sum of the numbers from the chart. 	Select some numbers from a 1-100 number chart and think about an efficient way to calculate their sum.	"Why can the teacher find the sum of 9 numbers so quickly?"
2 (select one)	A: Students will be able to explain that the same thinking approach of moving the numbers around can be applied to different number charts such as a calendar.	Explore the thinking approach discussed in Lesson 1 by creating problems that use other number charts such as a calendar and the multiplication table.	Can we use the same approach to with other number charts?
	B: Students will be able to explain that the same thinking approach of moving the numbers around can be applied even if we select the numbers by using something other than a 3x3 array.	Select numbers from a number chart differently such as 3x5 or 5x7 arrays, and create problems.	Can we use the same approach even if we select the number differently?

Lesson 2: Anticipated students' responses

Sun	Mon	Tue	Wed	Th	Fri	Sat
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
29	30	31				

	1	2	3	4	5	6	7	8	
1	1	2	3	4	5	6	7	8	
2	2	4	6	8	10	12	14	16	
3	3	6	9	12	15	18	21	24	
4	4	8	12	16	20	24	28	32	
5	5	10	15	20	25	30	35	40	
6	6	12	18	24	30	36	42	48	
7	7	14	21	28	35	42	49	56	
8	8	16	24	32	40	48	56	64	

11	12	13	14	15
21	22	23	24	25
31	32	33	34	35

		13	14	15		
		22	23	24	25	26
31	32	33	34	35	36	37
		42	43	44	45	46
			53	54	55	

Today's lesson

(1) Goals

- Students will be able to calculate the sum of numbers selected from a hundred chart by developing a strategy to move numbers based on patterns in the chart.
- Students will find the number chart intriguing and fun as they calculate the sum of the numbers from the chart.

(2) Flow of the lesson

	Main problems and anticipated responses Learning tasks	Evaluation (○)/Support (●) Points of consideration (◎)																									
Understanding Task	<p>1. Understanding the task</p> <p>T: I am going to pick 9 numbers from the chart that are arranged in a 3 x 3 array. Let's have a race who can find the sum first.</p> <p>Example</p> <table style="margin-left: 40px;"> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td></tr> <tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td></tr> <tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td></tr> </table> <p>GT: I got it! C: Already? You aren't even using a calculator.</p> <p>Repeat 2~3 times.</p> <p>C: How can he calculate so quickly? C: What kind of computation is he doing? C: There must be a secret to find the total of the 9 numbers inside the square.</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Let's come up with a strategy to calculate the total of the 9 numbers efficiently and explain the strategy.</p> </div>	11	12	13	14	15	21	22	23	24	25	31	32	33	34	35	41	42	43	44	45	51	52	53	54	55	<p>◎ Prepare a 3 x 3 array outline to match the number chart so that it can be placed on the chart.</p> <p>◎ Students will use calculators to find the sum. A guest teacher will calculate the sum mentally.</p> <p>◎ After children calculated the sum, check that GT's answer was indeed correct.</p>
11	12	13	14	15																							
21	22	23	24	25																							
31	32	33	34	35																							
41	42	43	44	45																							
51	52	53	54	55																							
Individual problem solving	<p>2. Individual problem solving</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>A Separate the ones and the tens digits $(10 + 20 + 30) \times 3 = 180$ $(1 + 2 + 3) \times 3 = 18$ $180 + 18 = 198$</p> </div>	<p>◎ After the task is posed, the teacher will circulate among the students and provide the necessary one-on-one support. For those who do not understand, provide a small group learning opportunity.</p> <p>A</p> <p>○ Students identify equal size numbers by separating the ones and tens digits.</p> <p style="text-align: center;">↓</p> <p>● Help students recognize the thinking approach of creating equal numbers by evening out the numbers.</p>																									

B Combine a small number with a large number.

11 12 13

21 22 23

31 32 33

$(11+33)+(12+32)+(13+31)+(21+23)+22$ ---

44 44 44 44 middle #

$$44 \times 4 + 22 = 198.$$

C Average each row to find the total

12 12 12

22 22 22

32 32 32

$$12 \times 3 + 22 \times 3 + 32 \times 3 = 198$$

D Average each column to find the total.

21 22 23

21 22 23

21 22 23

$$21 \times 3 + 22 \times 3 + 23 \times 3 = 198$$

E Average all numbers to find the total

22 22 22

22 22 22

22 22 22

$$22 \times 9 = 198$$

3. Share and critique solutions

T: I saw that someone solved the problem by using the math sentence,
 $(10 + 20 + 30) \times 3 + (1 + 2 + 3) \times 3$.
 How did this person think about the problem?

B

○ Students try to make the same number by combining a large number with a small number.



● Guide the students to the idea that by evening out the number obtained by combining 2 numbers we can change all 9 numbers into 22.

C & D

○ Students try to make the same number by averaging.



● Help students realize that by extending the idea to average the resulting columns or rows, they can even out all numbers to 22.

E

○ Students recognize that all numbers can be averaged to make the middle number, 22.

◎ Show only the math sentence and ask students to interpret the thinking represented by the sentence.

Summary	<p>C1 S/he is separating the ones and the tens digits in each number. If you look at the tens digits in column, it is always (10 + 20 + 30). If you look at the ones digits in each column, it is always (1 + 2 + 3). Since there are 3 columns, $\times 3$.</p> <p>T: How about the math sentence, $44 \times 4 + 22$?</p> <p>C2 If you add 11 and 33, 12 and 32, 13 and 31, and 21 and 23, we have four 44's. Then add the remaining 22.</p> <p>T: How about the math sentence, $12 \times 3 + 22 \times 3 + 32 \times 3$?</p> <p>C3 I don't see three 12's anywhere in the chart. Where are they coming from?</p> <p>T: I agree. We don't see three 12's in this chart. Maybe we need to do something to the numbers.</p> <p>C4 If you look at the numbers in the first row, 11, 12, and 13, we can make all of them 12 if we move 1 from 13 to 11.</p> <p>C5 We can make all numbers in the second row 22, and the third row 32.</p> <p>C6 We can use the middle number in each row.</p> <p>T: How about this math sentence, $21 \times 3 + 22 \times 3 + 23 \times 3$?</p> <p>C7 This time, we are looking at columns. If you move 10 from 31 in the first column, we can make 21.</p> <p>C8 We are using the middle numbers again.</p> <p>T: I also saw some people using the math sentence, 22×9. How were they thinking?</p> <p>C9 They combined C and D together.</p> <p>C10 You can even out each row as in C. Then, you can even out each column. We can make all numbers the same as the middle number, 22.</p> <p>4. Summarize</p> <p>T: So, which idea do you think I was using?</p> <p>C11 It must be E because it is the easiest and the shortest.</p>	<ul style="list-style-type: none"> ● Prepare several copies of the number chart so that children can use the chart freely to explain their ideas. ● For those numbers that are not on the number chart, make sure students understand how they were derived. ● Have students compare C & D so that they can recognize that they are using the same idea. Before sharing E, provide time for more students to come up with the idea on their own. ● Make sure that students understand that GT was using $22 \times 9 = 198$ to find the sum.
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