

Mathematics Lesson Plan

Date: June 25 (Mon), 2007, Period 2 (9:45 – 10:35)
Class: Class C, Grade 7, 42 students (21 boys, 21 girls)
Room: Mathematics Room
Teacher: Nobuko Nakamoto

1. Name of the unit: Letters and mathematical expressions

2. Overview of the unit

In this unit, students will learn to use letters in place of numbers and quantities as appropriate. They will learn to represent relationships and properties among quantities generally and concisely by using letters. Being able to interpret mathematical expressions involving letters is an important focus as well. Students will also develop procedural fluency in manipulating mathematical expressions appropriately to suite the purposes. It should be kept in mind that students have not encountered mathematical expressions with letters in elementary schools. Therefore, its introduction should be approached carefully.

2.1 Goals of the unit

Students will understand the roles of letters in mathematical expressions and be able to use mathematical expressions with letters appropriately to suite the purposes. Moreover, students will understand how to calculate mathematical expressions with letters and be able to perform such calculation.

1. Students will be able to represent various numbers and quantities using mathematical expressions with letters and interpret the meanings of such expressions.
2. Students will be able to multiply or divide linear expressions and also add and subtract linear expressions.

2.2 Relationship to elementary school mathematics

In elementary school, students learn how to represent properties and relationships of numbers and quantities concisely using math sentences and mathematical expressions with words by 4th grade. They also learn to interpret math sentences and use formulas as appropriate. In 5th grade, students learn how to

investigate the relationship among quantities when the relationship is given in a math sentence. Finally, in 6th grade, students learn to represent quantitative relationships in math sentences. However, they do not study mathematical expressions with letters.

In 7th grade, students will learn how to represent relationships and properties of numbers and quantities using mathematical expressions with letters. Moreover, they will learn how to interpret and manipulate such expressions. Through this study, students are to learn the merits of using letters in mathematical expressions. To help students overcome their hesitancy toward the use of letters in mathematical expressions and deepen their understanding gradually, we will approach the idea of letters as generalized numbers carefully through activities such as substituting various numbers in formulas or mathematical expressions with words.

3 Students' current state of learning

Some students are very much interested in mathematics while others feel very weak in mathematics. Thus, students' mathematical abilities vary widely. Although we cannot say that the students actively participate in class discussion, their attitudes toward mathematics lessons are generally positive.

4. Textbook

Mathematics, Lower Secondary School Grade 1, New Edition. (Dainippon Tosho)

5. Materials

Square cards (for display and for students' individual use)

6. Plan of instruction

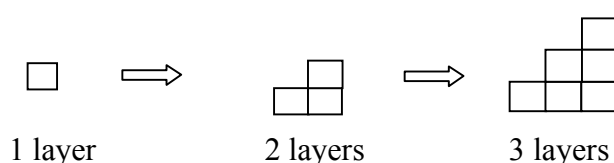
Sec.	Topic	Lessons	Contents
1. Letters and Expressions	1. Mathematical expressions with letters	1	<ul style="list-style-type: none"> • Meanings of mathematical expressions with letters • Representing numbers and quantities using letters
	2. Conventions on the use of letters	1	<ul style="list-style-type: none"> • How to represent quotients using letters • Following conventions to write mathematical expressions with letters
	3. Representing numbers and quantities using mathematical expressions	1	<ul style="list-style-type: none"> • Representing numbers and quantities using mathematical expressions
	4. Interpreting what mathematical expressions represent	2	<ul style="list-style-type: none"> • Interpreting what mathematical expressions represent Today's lesson is the 1 st of two lessons.
	5. Values of mathematical expressions	1	<ul style="list-style-type: none"> • Values of letters and values of mathematical expressions and how to calculate them
2. Calculation of expressions	1. Linear expressions and their terms	1	<ul style="list-style-type: none"> • Meanings of terms, coefficients and linear expressions • Calculation of simple linear expressions
	2. Multiplying linear expressions by numbers	1	<ul style="list-style-type: none"> • Multiplying linear expressions by numbers
	3. Dividing linear expressions by numbers	1	<ul style="list-style-type: none"> • Dividing linear expressions by numbers
	4. Addition and subtraction of linear expressions	1	<ul style="list-style-type: none"> • Addition and subtraction of linear expressions • Calculation of various linear expressions
	◎ Practices	1	
Problems for Chapter 2		1	

8. Previous lessons

As students study mathematical expressions, there are three foci—representing with mathematical expressions, manipulating mathematical expressions, and interpreting mathematical expressions. To help students understand that representing and interpreting mathematical expressions are types of communication activities, students were given a task which encouraged them to interpret mathematical expressions geometrically.

Task

One-centimeter squares were arranged in the manner shown below. Write a mathematical expression for the perimeter of the figure when there are n layers of squares.



As students engaged in this task, they determined the perimeter of the figures with 1 layer → 2 layers → 3 layers of squares:

$$1 \text{ layer} \text{ ----- } 1 \times 4 = 4$$

$$2 \text{ layers} \text{ ----- } 1 \times 8 = 8$$

$$3 \text{ layers} \text{ ----- } 1 \times 12 = 12$$

From these, students inductively concluded that

$$n \text{ layers} \text{ ----- } 1 \times 4n = 4n.$$

To help students understand why in this expression, $4n$, the perimeter of the figure is 4 times the number of layers, n , students were encouraged to interpret the situation geometrically as follows.

By moving the sides of the squares as shown in the figure below, we can see that the perimeter of the figure is the same as the perimeter of one large square. Since the length of one side of the large square is the same as the number of layers, n , the perimeter of the figure is 4 times of n , or $4n$.




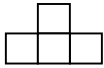
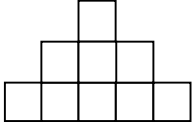
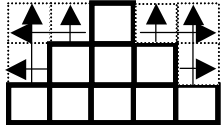
9. Goals of this lesson

In today's lesson, continuing from the previous lesson, students will examine the relationship between quantities inductively and express this relationship generally by using mathematical expressions with letters. Furthermore, interpreting the

mathematical expression will become the focus of the lesson.

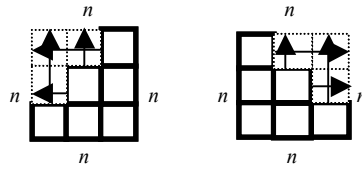
Students will be encouraged to represent the relationship they find in geometric drawings with mathematical expressions, and to relate the drawings and the mathematical expressions to each other. By making connections between the manipulation of geometric drawings and the manipulation of mathematical expressions, students will realize that mathematical expressions represents specific meanings. Moreover, they will recognize the importance of representing with and interpreting mathematical expressions, as well as the merits of representing relationships using mathematical expressions with letters. These are the goals of this lesson.

10 Flow of the lesson

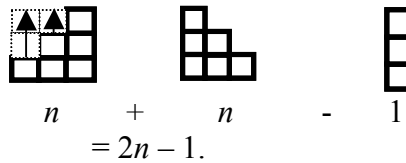
Step Time	Purposes Questions	Anticipated responses	Points of consideration
Intro. 5 min	<ul style="list-style-type: none"> • Pose the task <p>T:</p>	<p>Task 1</p> <p>One-centimeter squares were arranged in the manner shown below. Write a mathematical expression with letters for the perimeter of the figure when there are n layers of squares.</p> <p>1 layer </p> <p style="text-align: center;">↓</p> <p>2 layers </p> <p style="text-align: center;">↓</p> <p>3 layers </p>	<ul style="list-style-type: none"> • Prepare square cards and actually arrange them. • Encourage Ss to relate to what they discussed in the previous lesson.
Develop (1) 20 min.	<ul style="list-style-type: none"> • Understanding the task <p>T: Do you understand what the task is asking? How is this similar to the problem we did in the previous lesson?</p> <ul style="list-style-type: none"> • Solve the problem <p>T: How can we solve this problem?</p> <p>T: How can you represent the perimeter of the figure with the letter n?</p>	<p>S: We are arranging squares systematically.</p> <p>S1: Construct a table showing the number of layers and the perimeter of the figure. From the table, the pattern between the number of layers and the perimeter of the figure is identified.</p> <p>S2: Similar to what they did in the previous day's lesson, try to move the sides of the small squares, as shown below. When the sides are moved, the sides of the squares that form the figure can be made into a large rectangle.</p>  <p>Relate to what they did in the previous day's lesson.</p>	<ul style="list-style-type: none"> • If students cannot get started, ask them to think about the problem from the previous lesson. • Help them realize that a table may be useful to identify patterns.

T: How did you obtain these expressions?
Please explain using mathematical expressions.

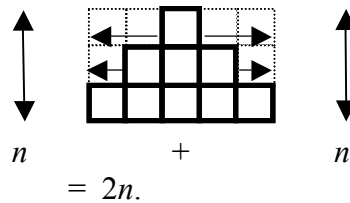
T: Please explain so that the relationship between the manipulation of the geometric figures and the mathematical expressions is clear.



The length across is:



The vertical lengths are:



Therefore, the total length across the figure is,

$$\begin{aligned} & 2 \times (2n - 1) + 2n \\ &= 2(2n - 1) + 2n \\ &= 4n - 2 + 2n \\ &= 6n - 2 \end{aligned}$$

Layers	1	2	3	4	...
Length around	4	10	16	22	...

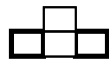
$$+ 6 \quad + 6$$

S2: Trying to identify the pattern from the table.

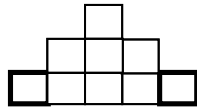
S2: Every time the number of layers increases by 1, the perimeter of the figure increases by 6 cm.

Explain this idea using diagrams.

- Have students represent with mathematical expressions.
- Have students think about the relationship between the manipulation of the geometric figures and the mathematical expressions.



2 squares x 3 sides
(3 x 2)



2 squares x 3 sides
(3 x 2)

The arrangement with n layers will have $(n - 1)$ more squares with 3 sides each, or 6 cm , than the first square, 4 cm . The total length around the figure can be represented with the following mathematical expression:

$$4 + 6(n - 1) = 6n - 2.$$

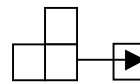
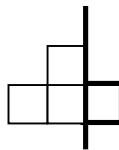
S3: Trying to identify the pattern from a table.

Layers	1	2	3	4	...
Length around	4	10	16	22	...

1 layer $4 = 1 \times 4$
 2 layers $10 = 2 \times 4 + \underline{2}$
 3 layers $16 = 3 \times 4 + \underline{4}$
 4 layers $22 = 4 \times 4 + \underline{6}$

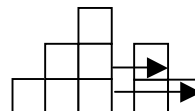
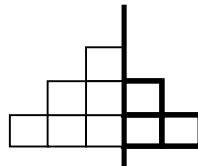
Explain this with diagrams.

2 layers



$$2 \times 4 + 2 \times 1$$

3 layers



$$3 \times 4 + 2 \times 2$$

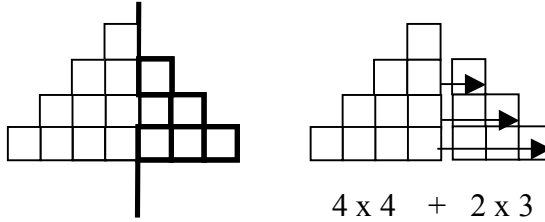
Develop 2
20 min.

- Pose the task
T: This time, we would like to find the number of squares when there are n layers.

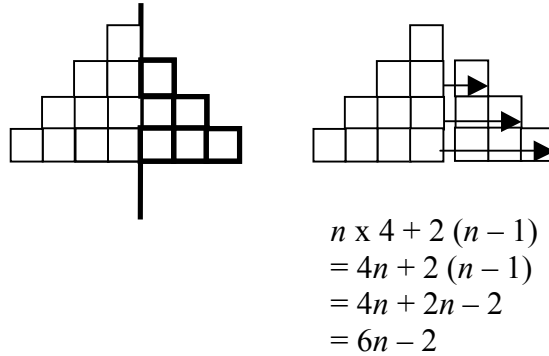
- Understand the task
T: How can we think about this problem?

T: How would you write a mathematical expression based on this table?

4 layers



From these, think about n layers.



$$\begin{aligned} n \times 4 + 2(n - 1) \\ = 4n + 2(n - 1) \\ = 4n + 2n - 2 \\ = 6n - 2 \end{aligned}$$

Task 2

Write a mathematical expression that represents the number of squares in a n -layered arrangement.


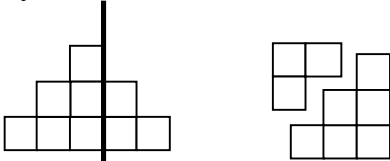
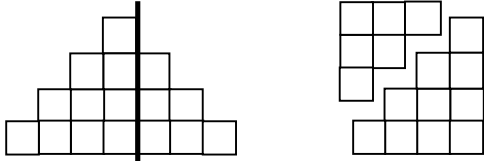
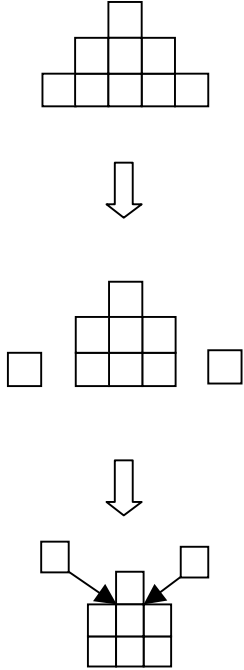
S3 Identify the pattern from a table.

Layers	1	2	3	4	...
Squares	1	4	9	16	...

S: The number of squares in each layer is:

1 layer $1 = 1^2$ (squares)

- Have the student represent the pattern using a mathematical expression.

	<p>T: If the number of squares in an n-layered arrangement is n^2, what is this expression representing?</p> <p>T: Let's think about why we get the area of a square.</p>	<p>2 layers $4 = 2^2$ (squares) 3 layers $9 = 3^2$ (squares) 4 layers $16 = 4^2$ (squares) So the number of squares when there are n layers must be n^2.</p> <p>S: The area of a square with n units on each side.</p> <p>S: (Explains using diagrams below.)</p> <p>2 layers</p>  $(1 + 2) + 1$ $\begin{array}{r} 1 \\ +) 1 + 2 \\ \hline 2 + 2 = 2 \times 2 = 2^2 \end{array}$ <p>We can make a square by cutting and rearranging .</p> <p>3 layers</p>  $(1 + 2 + 3) + (2 + 1)$ $\begin{array}{r} 2 + 1 \\ +) 1 + 2 + 3 \\ \hline 3 + 3 + 3 = 3 \times 3 = 3^2 \end{array}$ <p>4 layers</p>  $(1 + 2 + 3 + 4) + (3 + 2 + 1)$ $\begin{array}{r} 3 + 2 + 1 \\ +) 1 + 2 + 3 + 4 \\ \hline 4 + 4 + 4 + 4 \\ = 4 \times 4 = 4^2 \end{array}$	<ul style="list-style-type: none"> • Have students record in such a way that the correspondence between the manipulation of the geometric figures and the expressions can be easily seen. • Have students think about whether or not the explanation with diagrams can be generalized. <p>Note: The method below can make squares, but it cannot be generalized.</p> 
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11. Domains of evaluation

Interest, desire, and attitude toward mathematics	Mathematical ways of observing and thinking	Mathematical representations and procedures	Knowledge and understanding of numbers, quantities and geometric figures
<ul style="list-style-type: none"> • Ss will show their interest in the necessity and merits of using letters to represent relationships and properties of numbers and quantities. Ss will also willingly try to use and interpret mathematical expressions with letters. • Ss can extend their thinking through conjecturing and simplifying in the course of problem solving. 	<ul style="list-style-type: none"> • Ss will be able to represent relationships and properties among numbers and quantities in phenomena using letters, and think about them generally. • Ss will be able to identify numbers and quantities from mathematical expressions. • Ss will understand that letters are used to stand for numbers, and they can also consider an expression as a number. 	<ul style="list-style-type: none"> • Ss will be able to represent relationships and properties among numbers and quantities among phenomena using mathematical expressions with letters, and interpret mathematical expressions. 	<ul style="list-style-type: none"> • Ss will understand that, by using letters, we can represent relationships and properties among numbers and quantities generally, and we can also interpret such relationships and properties from mathematical expressions. • Ss will understand that mathematical expressions with letters represent the methods of calculation and their answers. • Ss will understand the conventions for using letters in mathematical expressions.

12. References

- 中学校学習指導要領解説編（数学）(Elaboration on the Course of Study for Lower Secondary School Mathematics). Ministry of Education, 2002.
- 観点別学習状況の新評価規準表（平成14年度版中学校数学）(Evaluation Standards for Assessment Domains: Lower Secondary School Mathematics). Kitao et. al. (Eds.), 2002.